The Art and Science of Decision-Making Under Severe Uncertainty

Moshe Sniedovich

Department of Mathematics and Statistics
The University of Melbourne
www.ms.unimelb.edu.au/∼moshe/

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More suitable title

Moshe in the Lions' Den

Abstract

Abstract

This presentation will give you a gentle guided tour of the art and science of decision-making Under Severe Uncertainty, covering the last 400 years or so.

The main objective is to explain the motivation for my Worst-Case Analysis / Maximin Campaign.

We shall examine a popular reserve design problem as a case study.

Programme

A Simple Problem

Good morning Sir/Madam:

I left on your doorstep four envelopes. Each contains some money. You are welcome to open any one of these envelopes and keep the money you find there.

Please note that as soon as you open an envelope, the other three will automatically self-destruct, so think carefully about which of these envelopes you should open.

To help you decide what you should do, I printed on each envelope the possible values of the amount of money (in Australian dollars) you may find in it. The amount that is actually there is equal to one of these figures.

Unfortunately the entire project is under severe uncertainty so I cannot tell you more than this.

Good luck!

Joe.

So What Do You do?

Example

Envelope	*Possible Amount (Australian dollars)*
$E1$	$20, 10, 300, 786$
$E2$	$2, 40000, 102349, 5000000, 99999999, 56435432$
$E3$	$201, 202$
$E4$	200

Vote!

Modeling and Solution

- What is a decision problem ?
- How do we model a decision problem?
- How do we solve a decision problem?

Decision Tables

Think about your problem as a table, where

- rows represents decisions
- columns represent the relevant possible states of nature
- entries represent the associated payoffs/rewards/costs

Example

Env	*Possible Amount ($AU)*				
$E1$	20	10	300	786	
$E2$	2	4000000	102349	500000000	56435432
$E3$	201	202			
$E4$	200				

Classification of Uncertainty

Classical decision theory distinguishes between three levels of uncertainty regarding the state of nature, namely

- Certainty
- Risk
- Strict Uncertainty

In our discussion

$$\text{Strict Uncertainty} \equiv \text{Severe Uncertainty}$$

Severe Uncertainty

Classical decision theory offers two basic principles for dealing with situations involving severe uncertainty, namely

- Laplace's Principle (1825)
- Wald's Principle (1945)

Conceptually:

$$\text{Certainty} \ \overset{\text{Wald}}{\Longleftarrow} \ \boxed{\text{Severe Uncertainty}} \ \overset{\text{Laplace}}{\Longrightarrow} \ \text{Risk}$$

$$\Downarrow$$

Info-Gap

????

Laplace's Principle of Insufficient Reason (1825)

Assume that all the states are equally likely, thus use a uniform distribution function on the state space and regard the problem as decision-making under risk.

Laplace's Decision Rule

$$\max_{d\in\mathbb{D}} \int_{s\in S_d} r(s,d)\mu(s)\,ds \qquad \text{Continuous case}$$

$$\max_{d\in\mathbb{D}} \frac{1}{|S_d|} \sum_{s\in S_d} r(s,d) \qquad \text{Discrete case}$$

Wald's Maximin Principle (1945)

Assume that Mother Nature is playing against you, hence apply the worst-case scenario. This transforms the problem into a decision-making under certainty.

Wald's Maximin Rule

$$\overset{\text{You!}}{\max_{d\in\mathbb{D}}} \ \overset{\text{Mama}}{\min_{s\in S_d}} \ f(d,s)$$

Historical perspective:

The gods to-day stand friendly, that we may,
Lovers of peace, lead on our days to age!
But, since the affairs of men rests still incertain,
Let's reason with the worst that may befall.

William Shakespeare (1564-1616)
Julius Caesar, Act 5, Scene 1

Laplace vs Wald

Example

Env	*Possible Amount ($AU)*				
$E1$	20	10	300	786	
$E2$	2	4000	102349	50000	56435
$E3$	201	202			
$E4$	200				

Example

Env	*Possible Amount ($AU)*					*Laplace*	*Wald*
$E1$	20	10	300	786		279	10
$E2$	2	4000	10234	50000	56435	24134.2	2
$E3$	201	202				201.5	201
$E4$	200					200	200

Laplace vs Wald

Example

Env	*Possible Amount ($AU)*					*Laplace*	*Wald*
$E1$	20	10	300	786		279	10
$E2$	2	4000	10234	50000	56435	24134.2	2
$E3$	201	202				201.5	201
$E4$	200					200	200

Severe Uncertainty

Warning!

- For obvious reasons, methodologies for decision-making under severe uncertainty are austere.
- Challenging mathematical modeling issues.
- There are no miracles in this business.
- The essential difficulty is: how do you sample the uncertainty region?
- If you are offered a methodology that is too good to be true,... it is!

Severe Uncertainty

A room with a view ...

Info-Gap	*Decision Theory*
Robust Optimization	*Operations Research*

Info-Gap

- A relatively young theory (Ben-Haim [2001, 2006]).
- Claims to be new and radically different from all existing theories.
- Based on a probability-free uncertainty model.
- Does not represent the state of the art in decision theory.
- Fundamentally flawed: conceptually, methodologically and technically.
- An excellent example of how quickly things can go wrong.
- Based on a very rigid mathematical modeling paradigm.
- The University of Melbourne seems to be one of its major international strongholds!

Generic Info-Gap Model

- An uncertainty region (set), $\mathfrak{U}$.
- A parameter u whose true value, u°, is unknown except that $u^\circ \in \mathfrak{U}$.
- An estimate, $\tilde{u} \in \mathfrak{U}$, of u°.
- A parametric family of nested regions of uncertainty, $\mathcal{U}(\alpha, \tilde{u}) \subseteq \mathfrak{U}, \alpha \geq 0$, of varying size ($\alpha$), centered at $\tilde{u}$. It is assumed that $\mathcal{U}(0, \tilde{u}) = \{\tilde{u}\}$ and that $\mathcal{U}(\alpha, \tilde{u})$ is non-decreasing with α, namely

$$\alpha'', \alpha' \in \mathbb{R}_+, \;\; \alpha'' > \alpha' \Longrightarrow \mathcal{U}(\alpha', \tilde{u}) \subseteq \mathcal{U}(\alpha'', \tilde{u}) \tag{1}$$

- Set of decisions available to the decision maker, $\mathbb{Q}$.
- A real-valued reward function, R, on $\mathbb{Q} \times \mathfrak{U}$.
- A critical reward level, $r_c \in \mathbb{R}$.

Generic Info-Gap Model

Robustness of a decision

$$\hat{\alpha}(q, r_c) := \max \left\{ \alpha : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \tag{2}$$

Optimal robustness

$$\hat{\alpha}(r_c) : = \max_{q \in \mathbb{Q}} \ \hat{\alpha}(q, r_c) \tag{3}$$

$$= \max_{q \in \mathbb{Q}} \ \max \left\{ \alpha : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \tag{4}$$

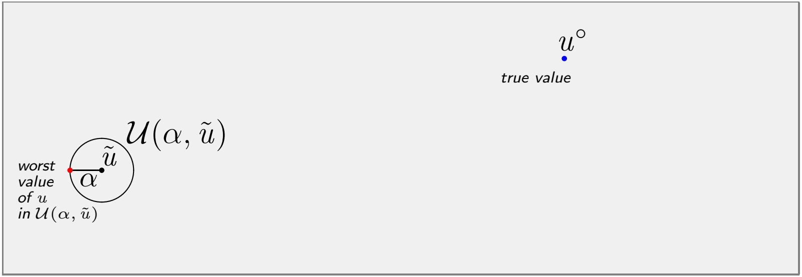
Generic Info-Gap Model

Complete Generic Model

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \ \hat{\alpha}(q, r_c) \tag{5}$$

$$= \max_{q\in\mathbb{Q}} \ \max \left\{ \alpha : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\} \tag{6}$$

Region of Severe Uncertainty, $\mathfrak{U}$



Fundamental FAQs

Complete Generic Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \tag{7}$$

Fundamental FAQs

1. Is this new? Definitely not!
2. Is this radically different? Definitely not!
3. Does it make sense? Definitely not!

A Room with a view . . .

Info-Gap

Decision Theory

Robust Optimization

Operations Research

Myths and Facts

Myth # 1

Info-Gap is a new theory that is radically different from all other theories for decision-making under severe uncertainty (Ben-Haim [2001, 2006])

Fact # 1

Info-Gap is a simple instance of Wald's Maximin model (see formal proof in Sniedovich [2006])

Maximin	*Info-Gap*
$\max_{d\in\mathbb{D}} \min_{s\in S_d} f(d,s)$	$\max_{q\in\mathbb{Q}} \max \left\{\alpha : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\}$

Myths and facts

Conventional format:

$$\alpha(r_c) := \max_{q\in\mathbb{Q}} \max \left\{ \alpha : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\}$$

Standard Maximin format:

$$z^*(r_c) := \max_{q\in\mathbb{Q},\alpha\geq 0} \min_{u\in\mathfrak{U}(\alpha,\tilde{u})} \overbrace{\alpha(r_c \preceq R(q,u))}^{f(q,\alpha,u)}$$

$$a \preceq b := \begin{cases} 1 & , \quad a \leq b \\ 0 & , \quad a > b \end{cases}$$

Myth and Facts

Theorem

Info-Gap's generic model is neither new nor radically different. It is a simple instance of Wald's [1945] famous Maximin model.

Proof

See above.

Myths and Facts

Myth # 2

Info-Gap generates robust solutions for decision-making problems under severe uncertainty.

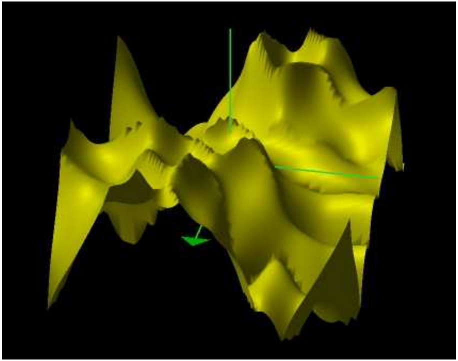
Fact # 2

There is no reason to believe that under severe uncertainty the solutions generated by Info-Gap are robust (see explanation and counter examples in Sniedovich [2006]).

Region of Severe Uncertainty, $\mathfrak{U}$

Myths and Facts

Note the difference between local and global optimization.



Myths and Facts

Myth # 3

It is better to satisfice than to optimize.

Fact # 3

Any satisficing problem can be formulated as an (equivalent) optimization problem. See formal proof in Sniedovich [2006].

Comments:

- Strictly and bluntly speaking, the assertion that satisficing is superior to optimizing is pure nonsense.
- What is important is what you optimize and what you satisfice.

Myths and facts

Theorem

Any satisficing problem can be expressed as an (equivalent) optimization problem.

Proof.

Let I denote the universal indicator function:

$$I_Z(\xi) := \begin{cases} 1 & , \quad \xi \in Z \\ -\infty & , \quad \xi \notin Z \end{cases}$$

Then clearly,

$$\{x \in X \text{ and } y \in Y\} \Longleftrightarrow (x, y) = \arg\max_{x,y} \min \{I_X(x), I_Y(y)\}$$

$\square$

Myths and Facts

Myth # 4

Robust optimization is a contradiction in terms.

Fact # 4

Robust optimization is a well established area of optimization theory: more than 30-year old, and going strong!

Myths and Facts

Myth # 5

Info-Gap region of uncertainty is unbounded, therefore there is no worst case (Ben-Haim [2005]).

Fact # 5

This is pure nonsense.

Comments:

- There could be a worst case even if the region of uncertainty is unbounded.
- There is a worst case in all problems where Info-Gap yields a solution (Sniedovich [2006]).

Myths and Facts

620-161: Introductory Mathematics

The most classical saddle point on Planet Earth is associated

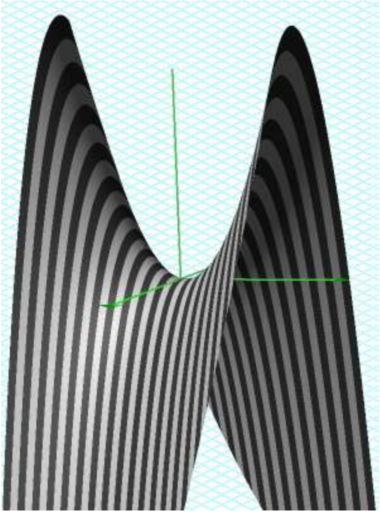
with the unbounded region $\mathbb{R}^2$ and the function

$$f(x, y) := x^2 - y^2$$

The saddle point is the solution to the Maximin problem

$$z^* := \max_{y \in \mathbb{R}} \min_{x \in \mathbb{R}} \left\{ x^2 - y^2 \right\}$$

$$z^* := \max_{y \in \mathbb{R}} \min_{x \in \mathbb{R}} \left\{ x^2 - y^2 \right\}$$



Myths and Facts

Ben-Haim [2001-2006] confuses a number of aspects of the Info-Gap uncertainty model:

$$\alpha(r_c) := \max_{q\in\mathbb{Q}} \max \left\{ \alpha : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\}$$

- α is unbounded.
- $\mathcal{U}(\alpha,\tilde{u})$ is unbounded.
- $R(q,u)$ is unbounded.

Example (Ben-Haim [2006])

- $\mathcal{U}(\alpha,\tilde{u}) := \left\{ u \in [0,1] : \left| \frac{u-\tilde{u}}{\tilde{u}} \right| \leq \alpha \right\} \ , \ \alpha \geq 0$
- α is unbounded.
- $\mathcal{U}(\alpha,\tilde{u}) \subseteq [0,1]$ is bounded.
- There is definitely a worst case!

Myths and Facts

Theorem

Info-Gap's uncertainty model is subject to a worst case.

Proof.

$$\hat{\alpha}(q,r_c) := \max_{\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \alpha\cdot(r_c \preceq R(q,u)) \tag{8}$$

$$= \max_{\alpha\geq 0} G(\alpha)\cdot H(q,\alpha) \tag{9}$$

where

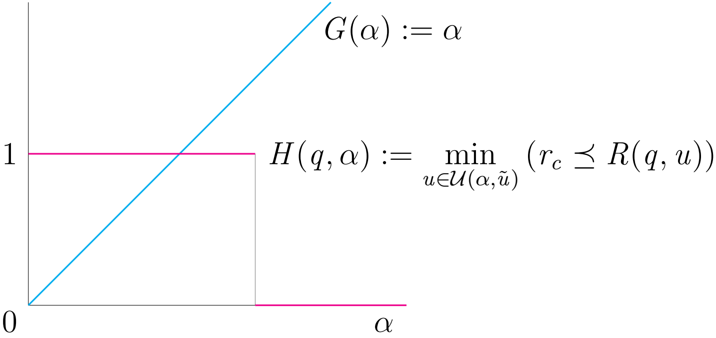
$$G(\alpha) := \alpha \ , \ \alpha \geq 0 \tag{10}$$

$$H(q,\alpha) := \min_{u\in\mathcal{U}(\alpha,\tilde{u})} (r_c \preceq R(q,u)) \ , \ \mathbb{Q}, \alpha \geq 0 \tag{11}$$

Clearly, $G(\alpha)\cdot H(q,\alpha) \in \{0,\alpha\}$. □

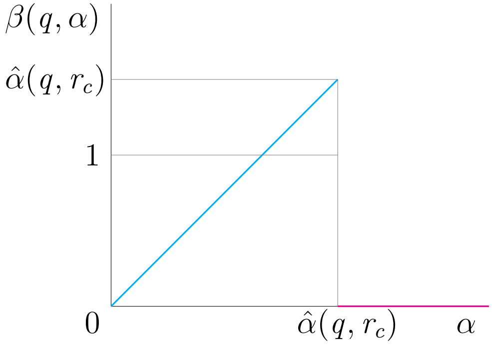
Myths and Facts

$$\hat{\alpha}(q, r_c) := \max_{\alpha \geq 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \alpha \cdot (r_c \preceq R(q, u))$$



Myths and Facts

$$\beta(q,\alpha) := G(\alpha)\cdot H(q,\alpha) = \alpha \cdot \min_{u\in\mathcal{U}(\alpha,\tilde{u})} (r_c \preceq R(q,u))$$
$$\in \{0,\alpha\}$$



Myths and Facts

Myth # 6

Info-Gap deals with severe uncertainty.

Fact # 6

Info-Gap does not deal with severe uncertainty. It ignores it.

This involves:

- Replacing severe uncertainty by a poor estimate of the parameter under consideration.
- Conducting standard maximin analysis in the neighborhood of this estimate.

Correction:

Info-Gap Interpretation

$\hat{\alpha}(q, r_c) :=$ robustness of decision q given the required reward r_c.

Correct interpretation

$\hat{\alpha}(q, r_c, \tilde{u}) :=$ robustness of decision q given the required reward r_c, in the neighborhood of the POOR estimate $\tilde{u}$ that is likely to be SUBSTANTIALLY WRONG.

Myths and Facts

Myth # 7

Info-Gap is a methodology for decision-making under severe uncertainty.

Fact # 7

Practicing Info-Gap amounts to voodoo decision-making:

1. Replacing severe uncertainty by a poor estimate of the parameter under consideration.
2. Conducting standard maximin analysis in the neighborhood of this estimate.

Voodoo

Encarta online Encyclopedia

Voodoo n

1. A religion practiced throughout Caribbean countries, especially Haiti, that is a combination of Roman Catholic rituals and animistic beliefs of Dahomean enslaved laborers, involving magic communication with ancestors.
2. Somebody who practices voodoo.
3. A charm, spell, or fetish regarded by those who practice voodoo as having magical powers.
4. A belief, theory, or method that lacks sufficient evidence or proof.

Voodoo Decision-Making

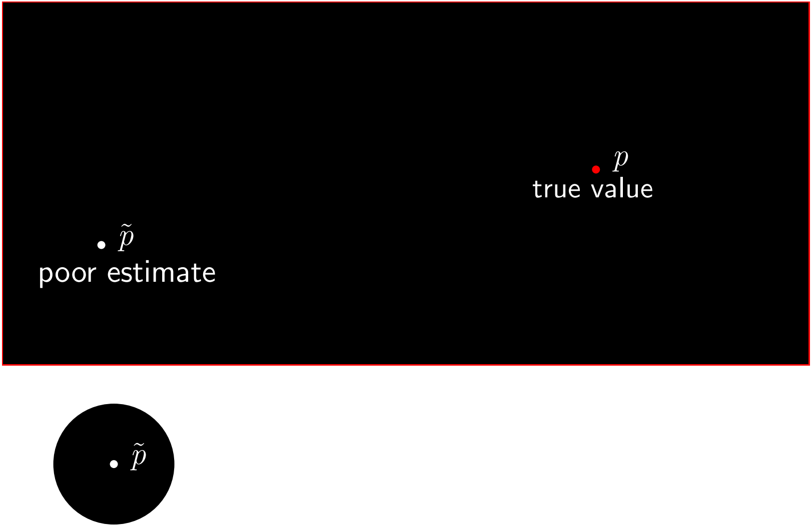
Early reference,

> *Richard Tam, a visionary and entrepreneur, started iUniverse he once told me after seeing how major publishing companies deal in false scarcity and voodoo decision-making processes. "They don't know where – or who – their customers are. They have to find them all over again every time they need to market something new."*
>
> *Wednesday, August 28, 2002*
>
> http://blogs.salon.com/0001111/2002/08/28.html

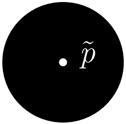
Conclusions

- Decision-making under severe uncertainty is difficult.
- This is a very active area of research/practice.
- The Robust Optimization literature is very relevant.
- The Operations Research literature is very relevant.
- The Decision Theory literature is very relevant.
- Info-Gap is fundamentally flawed and is not suitable for decision-making under severe uncertainty.
- Info-Gap exhibits a severe information-gap about the state of the art in decision-making under severe uncertainty.

A thing to remember



A thing to remember



Off the record

The Ten Natural Laws of Operations Analysis

1. Ignore the problem and go immediately to the solution, that is where the profit lies.
2. There are no small problems only small budgets.
3. Names are control variables.
4. Clarity of presentation leads to aptness of critique.
5. Invention of the wheel is always on the direct path of a cost plus contract.
6. Undesirable results stem only from bad analysis.
7. It is better to extend an error than to admit to a mistake.
8. Progress is a function of the assumed reference system.
9. Rigorous solutions to assumed problems are easier to sell than assumed solutions to rigorous problems.
10. In desperation address the problem.

Bob Bedow, *Interfaces* 7(3), p. 122, 1979.

Off the record

Join the Worst Case / Maximin Campaign

www.ms.unimelb.edu.au/~moshe/

I Love

$$v^* := \max\limits_{d\in\mathbb{D}}^{\textit{me!}} \ \min\limits_{s\in S_d}^{\textit{mama!}} f(d, s)$$

Wald's Maximin Principle

Intermission

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